

Cyclotron Harmonic Waves in Warm Plasmas¹

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It has been predicted theoretically that cyclotron harmonic waves should propagate in warm plasma confined by a magnetic field, and show cutoff and resonance behavior associated with harmonics of the cyclotron frequency and the upper hybrid resonance frequency. In this paper, a simplified analysis based on the quasi-static approximation is presented and used to establish the relevant dispersion relations. Computer solutions of these are presented showing the influence of the magnetic field and discharge parameters on the propagation. A number of recent experimental results can be explained in terms of this type of wave motion and serve to establish its validity. This work is discussed briefly. The paper discusses some of the implications of this mode in laboratory and ionospheric plasma physics.

1. Introduction

Wherever possible in theoretical plasma wave propagation studies the analysis is simplified by neglecting thermal motions of the charged particles. This allows the plasma to be treated as a medium possessing a frequency-dependent, effective permittivity, which may be a tensor quantity if a static magnetic field is present. This approach is familiar from the literature of ionospheric propagation and may easily be extended to include the effects of collisions [Ratcliffe, 1959; Budden, 1961].

It is recognized that the possible excitation of waves whose propagation depends on the existence of nonzero thermal velocities is ignored in the cold plasma approximation. To study these, an approach based on the Boltzmann equation is required. This may take two directions. The first possibility, the hydrodynamic approach, is to work with moments of the Boltzmann equation, and leads directly to a description of the plasma behavior in terms of macroscopic quantities such as charge density, current, and electron pressure. Analyses of this nature have led to the prediction of acoustic waves: At high frequencies these have the well-known plane-wave dispersion relation

$$\omega^2 \approx \omega_p^2 + k^2 \left(\frac{3\kappa T_e}{m} \right), \quad (1)$$

where propagation is as $\exp i(\omega t - \vec{k} \cdot \vec{r})$, ω_p is the electron plasma frequency, κ is Boltzmann's constant, T_e is the electron temperature, and m is the electronic mass. Although (1) has never been verified experimentally, there is very strong evidence from scattering experiments carried out in the laboratory with inhomogeneous cylindrical plasmas, and described accurately by the hydrodynamic theory [Crawford, 1963, 1964a; Nickel et al., 1963; Parker et al., 1964] that the expression is a valid approximation. We shall take up this point again shortly.

At low frequencies, electrostatic sound waves are predicted with phase velocity independent of charge density and frequency. For a plane wave, the dispersion relation is,

$$\omega \approx k \left(\frac{\kappa T_e}{M} \right)^{1/2}, \quad (2)$$

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where M is the ion mass. These waves have been observed in laboratory discharges [Crawford, 1961; Little, 1962].

The second method of employing the Boltzmann equation is to study individual particle trajectories, and then to integrate over velocity space to obtain macroscopic quantities. It was first pointed out by Landau [1946] that this procedure shows (1) to be incorrect. An additional damping term is required, due to trapped particles traveling with the wave [Bohm and Gross, 1949], which cannot be predicted by hydrodynamic theory. This so-called "Landau damping" becomes very strong at wavelengths comparable to a Debye length, and has been studied quantitatively recently in the laboratory [Malmberg and Wharton, 1964].

Since the single particle theory can be demonstrated to contribute damping terms to the acoustic modes predicted by hydrodynamic theory, a question now arises: does it also predict the existence of new types of propagating waves? The answer is that it does. Cyclotron harmonic waves can propagate in warm plasma immersed in a magnetic field. The objects of this paper are to demonstrate physically how these waves can occur; to give computed dispersion curves for them, and to indicate the nature of the very recent experimental evidence confirming their existence.

2. Theory

2.1. Single Particle Trajectory

First consider the dynamics of a single electron moving under the influence of a static magnetic field and a transverse rf electric field, E_{1x} , given by $E_x \exp i(\omega t - k_x x - k_z z)$. We have, for the unperturbed trajectory of figure 1,

$$\begin{aligned} v_R &= v'_R & v_z &= v'_z \\ \phi' &= \phi + \omega_c(t' - t), \end{aligned} \quad (3)$$

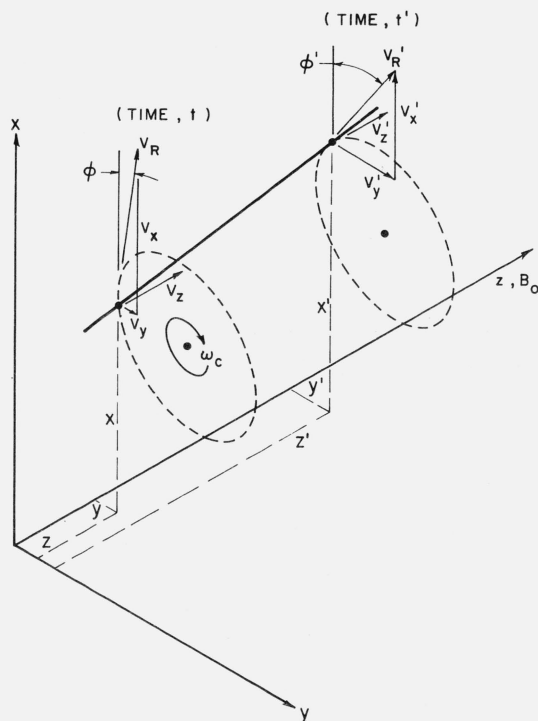


FIGURE 1. Single particle trajectory (the unperturbed motion is along a helical path, the projection of which in the x-y plane is a circle).

where ω_c is the electron cyclotron frequency. Some simple integration and trigonometric manipulation, leads to

$$\begin{aligned} z' &= z + v_z(t' - t) \\ x' &= x + \left(\frac{v_R}{\omega_c}\right) [\sin \{\omega_c(t' - t) + \phi\} - \sin \phi] \\ &= x + \left(\frac{2v_R}{\omega_c}\right) \sin \left[\frac{\omega_c}{2}(t' - t)\right] \cos \left[\frac{\omega_c}{2}(t' - t) + \phi\right]. \end{aligned} \quad (4)$$

For the x -component of the perturbed trajectory, x'_1 , we have the equation of motion,

$$\ddot{x}'_1 + \omega_c^2 x'_1 = \eta E_x \exp i(\omega t' - k_x x' - k_z z'), \quad (5)$$

where η is the electronic charge-to-mass ratio.

For small perturbations, the argument of the exponential may be replaced by its unperturbed value to obtain x'_1 correct to first order. This yields,

$$\begin{aligned} \ddot{x}'_1 + \omega_c^2 x'_1 &= A \exp i \left[(\omega - k_z v_z)(t' - t) - \left(\frac{k_x v_R}{\omega_c}\right) \sin \{\omega_c(t' - t) + \phi\} \right] \\ A &\equiv \eta E_x \exp i \left[\omega t - k_x \left(x - \left(\frac{v_R}{\omega_c}\right) \sin \phi\right) - k_z z \right], \end{aligned} \quad (6)$$

where we note that (v_R/ω_c) is the Larmor radius of the particle motion.

A well-known Bessel function identity may now be introduced [Bateman Manuscript Project, 1953]

$$\exp \left[-i \left(\frac{k_x v_R}{\omega_c}\right) \sin \{\omega_c(t' - t) + \phi\} \right] \equiv \sum_{n=-\infty}^{+\infty} J_n \left(\frac{k_x v_R}{\omega_c}\right) \exp (-in \{\omega_c(t' - t) + \phi\}). \quad (7)$$

Inserting this in (6) and integrating yields,

$$x'_1 = A \exp [i(\omega - k_z v_z)(t' - t)] \sum_{n=-\infty}^{+\infty} \frac{J_n \left(\frac{k_x v_R}{\omega_c}\right) \exp (-in \{\omega_c(t' - t) + \phi\})}{[\omega_c^2 - (\omega - k_z v_z - n\omega_c)^2]}. \quad (8)$$

The rf perturbation becomes very large when $(\omega - k_z v_z)$ approaches $(n \pm 1)\omega_c$, i.e., strong interactions occur between the rf electric field and the electron when the Doppler-shifted frequency approaches a cyclotron harmonic frequency. These cyclotron harmonic interactions result from two factors: the d-c rotational energy of the electron, and the fact that the rf electric field has a transverse variation ($k_x \neq 0$). This gives rise to space harmonics of the electric field over the direction of the electron motion, and hence strong interactions. As k_x or v_R decrease, the higher-order interactions will become progressively weaker. When $(k_x v_R/\omega_c)$ is zero only the fundamental cyclotron resonance predicted by cold plasma theory remains.

2.2. Multiparticle Treatment

The next step is to develop a multiparticle treatment in which the space-charge forces due to the presence of many electrons couple their motions and lead to a dispersion relation. The first attempt to do this appears to have been due to Malmfors [1950], who studied the stability of crossed-field microwave devices. His work contained an algebraic error that was corrected by Gross [1951], whose work was in turn taken up and extended by Sen [1952], using the Laplace transform approach

initiated by Landau [1946]. The subject was discussed again in a series of papers, the most complete of these being that due to Bernstein [1958] which corrected several errors in previous analyses.

In the averaging over the individual particle motions, two approaches are possible. A tensor expression for the rf current can be derived and substituted in Maxwell's equation [Bernstein, 1958; Stix, 1962] or an approximation can be made at the outset which allows us to work with scalar quantities and simplifies considerably the tedious integrations involved. This is the quasi-static, or "slow-wave," approximation, and consists in replacing the rf electric field \vec{E} , by the gradient of a potential, $(-\nabla\Phi)$. The approximation amounts to considering only waves for which

$$|k_x|^2 + |k_z|^2 \gg \frac{\omega^2}{c^2} |\epsilon_{ij}|, \quad (9)$$

where ϵ_{ij} are elements of the tensor permittivity of the plasma [Stix, 1962]. We assume, without loss of generality, variations in the x and z directions only, and c is the speed of light. It is also implied that \vec{E} and \vec{k} are parallel. Poisson's equation, for the rf space charge density is then,

$$-i(k_x E_x + k_z E_z) = (k_x^2 + k_z^2) \Phi = \frac{\rho_1}{\epsilon_0}. \quad (10)$$

To find a dispersion relation, it is now necessary to determine ρ_1 in terms of the rf electric fields in the system. The basic steps in the analysis will now be indicated without specifying the unperturbed electron velocity distribution.

2.3. Determination of the RF Charge Density

Liouville's theorem concerning particle number density, $f(x, y, z, v_x, v_y, v_z)$ in phase space states that the number density of particles in a volume element is conserved as this volume moves in phase space. Hence, if f_0 and f_1 are the d-c and rf components of f , we have

$$\frac{df_0}{dt} + \frac{df_1}{dt} = 0. \quad (11)$$

Written out in full, this is

$$\left[\frac{\partial f_0}{\partial t} + \vec{v} \cdot \nabla f_0 + \eta \vec{v} \times \vec{B}_0 \cdot \frac{\partial f_0}{\partial \vec{v}} \right] + \left[\frac{\partial f_1}{\partial t} + \vec{v} \cdot \nabla f_1 + \eta (\vec{E} + \vec{v} \times \vec{B}_0 + \vec{v} \times \vec{B}_1) \cdot \frac{\partial f}{\partial \vec{v}} \right] = 0. \quad (12)$$

To obtain f_1 correct to first order, we must solve

$$\left(\frac{df_1}{dt} \right)_0 = -\eta \vec{E} \cdot \frac{\partial f_0}{\partial \vec{v}}, \quad (13)$$

where the time-differentiation is along the unperturbed electron trajectory, and the rf magnetic field term can be neglected in a quasi-static analysis. Integration of (13) gives

$$f_1(t) = -\eta \int_{t_0}^t \vec{E}' \cdot \frac{\partial f_0}{\partial \vec{v}'} dt' + f_1(t_0). \quad (14)$$

Next, assume that f_1 is of the form $F \exp i(\omega t - k_x x - k_z z)$, and that \vec{E}' is given by $\vec{E} \exp i(\omega t' - k_x x' - k_z z')$. Substitution from (3) and (4) in (14) yields in cylindrical coordinates,

$$F(v_z, v_R, \phi) = -\eta \int_{-\infty}^t \exp i \left[(\omega - k_z v_z)(t' - t) - \left(\frac{2k_x v_R}{\omega_c} \right) \sin \left\{ \frac{\omega_c}{2} (t' - t) \right\} \cos \left\{ \frac{\omega_c}{2} (t' - t) + \phi \right\} \right] \\ \times \left[E_z \frac{\partial f_0}{\partial v_z} + E_x \frac{\partial f_0}{\partial v_R} \cos \{ \omega_c (t' - t) + \phi \} \right] dt'. \quad (15)$$

A lower limit of $t_0 = -\infty$ has been taken, assuming that there is no initial bunching of the electrons, i.e., $f_1(t_0) \rightarrow 0$ as $t_0 \rightarrow -\infty$. This is equivalent to assuming that a small loss component is present. The limit is also justifiable as the only one yielding periodic solutions in Φ [Stix, 1962].

To obtain ρ_1 , F must be integrated over v_z , v_R , and ϕ . The integration over ϕ is between limits of 0 and 2π and may be accomplished by means of the substitutions of τ for $(t' - t)$ and ψ for $[\phi + (\omega_c \tau/2)]$, and use of the Bessel function identity [Bateman Manuscript Project, 1953]

$$J_n \left(\frac{2k_x v_R}{\omega_c} \sin \frac{\omega_c \tau}{2} \right) \equiv \frac{i^n}{2\pi} \int_0^{2\pi} \exp \left[-i \left(\frac{2k_x v_R}{\omega_c} \sin \frac{\omega_c \tau}{2} \right) \cos \psi \right] \cos n\psi d\psi. \quad (16)$$

The resulting function, G , is given by

$$G(v_z, v_R) = -2\pi\eta \int_{-\infty}^0 \exp \left[i(\omega - k_z v_z)\tau \right] \left[E_z \frac{\partial f_0}{\partial v_z} J_0 \left(\frac{2k_x v_R}{\omega_c} \sin \frac{\omega_c \tau}{2} \right) \right. \\ \left. - iE_x \frac{\partial f_0}{\partial v_R} \cos \left(\frac{\omega_c \tau}{2} \right) J_1 \left(\frac{2k_x v_R}{\omega_c} \sin \frac{\omega_c \tau}{2} \right) \right] d\tau. \quad (17)$$

To complete the integration, a further Bessel function identity and its differential form are required [Bateman Manuscript Project, 1953]

$$J_0 \left(\frac{2k_x v_R}{\omega_c} \sin \frac{\omega_c \tau}{2} \right) \equiv \sum_{n=0}^{\infty} \Delta_n J_n^2 \left(\frac{k_x v_R}{\omega_c} \right) \cos n\omega_c \tau, \\ \cos \left(\frac{\omega_c \tau}{2} \right) J_1 \left(\frac{2k_x v_R}{\omega_c} \sin \frac{\omega_c \tau}{2} \right) \equiv \left(\frac{2\omega_c}{k_x v_R} \right) \sum_{n=1}^{\infty} n J_n^2 \left(\frac{k_x v_R}{\omega_c} \right) \sin n\omega_c \tau, \quad (18)$$

where $\Delta_0 = 1$; $\Delta_n = 2$, $n \neq 0$. Using these and again assuming that a slight loss component is present, we obtain

$$G(v_z, v_R) = -2\pi\eta\Phi \left[k_z \frac{\partial f_0}{\partial v_z} \sum_{n=0}^{\infty} \frac{\Delta_n (\omega - k_z v_z) J_n^2 \left(\frac{k_x v_R}{\omega_c} \right)}{(\omega - k_z v_z)^2 - (n\omega_c)^2} + \left(\frac{2\omega_c^2}{v_R} \right) \frac{\partial f_0}{\partial v_R} \sum_{n=1}^{\infty} \frac{n^2 J_n^2 \frac{k_x v_R}{\omega_c}}{(\omega - k_z v_z)^2 - (n\omega_c)^2} \right]. \quad (19)$$

An equivalent expression to this has been derived by Harris [1959, 1961]. Its value lies in the fact that when the unperturbed velocity distribution, f_0 , is specified, the rf charge density is immediately obtainable from

$$\rho_1 = e \int_{-\infty}^{+\infty} dv_z \int_0^{\infty} v_R G(v_z, v_R) dv_R. \quad (20)$$

If several species of particles are present their space-charge contributions are additive.

2.4. Dispersion Relations for Oblique Propagation ($k_x \neq 0$, $k_z \neq 0$)

It is instructive to consider first a uniform electron beam having a velocity distribution of the form

$$f_0 = \frac{n_0}{2\pi v_{R0}} \delta(v_R - v_{R0}) \delta(v_z - v_{z0}). \quad (21)$$

The δ -functions imply that the beam has a translational velocity, v_{z0} , and that all electrons have the same speed of gyration, v_{R0} , about the field lines. Substitution of (21) in (19) and (20), and combination of the result with (10) leads to the dispersion relation for the cyclotron harmonic waves on the beam,

$$0 = k_x^2 \left[1 - \omega_p^2 \sum_{n=1}^{\infty} \left\{ \frac{n^2 \frac{\partial}{\partial \mu} (J_n^2(\mu))}{\left(\frac{\mu}{2}\right) [(\omega - k_z v_{z0})^2 - (n\omega_c)^2]} \right\} \right] + k_z^2 \left[1 - \omega_p^2 \sum_{n=0}^{\infty} \Delta_n J_n^2(\mu) \left\{ \frac{(\omega - k_z v_{z0})^2 + (n\omega_c)^2}{[(\omega - k_z v_{z0})^2 - (n\omega_c)^2]^2} \right\} \right], \quad (22)$$

where ω_p is the electron plasma frequency corresponding to the beam density, and μ has been written for $(k_x v_{R0}/\omega_c)$. It will be noted that this is the parameter previously encountered in the single-particle model of (8). As $\mu \rightarrow 0$ we retrieve the Doppler-shifted cold-plasma dispersion relation

$$0 = k_x^2 \left[1 - \frac{\omega_p^2}{(\omega - k_z v_{z0})^2 - \omega_c^2} \right] + k_z^2 \left[1 - \frac{\omega_p^2}{(\omega - k_z v_{z0})^2} \right]. \quad (23)$$

Next, consider an isotropic Maxwellian electron velocity distribution such that

$$f_0 = n_0 \left(\frac{m}{2\pi\kappa T_e} \right)^{3/2} \exp \left[-\frac{m(v_z^2 + v_R^2)}{2\kappa T_e} \right]. \quad (24)$$

Substitution of (24) in (19) and (20) and combination with (10) will again lead to a dispersion relation. Alternatively, (22) may be regarded as the basic dispersion expression which is then written with (24) as

$$k_x^2 + k_z^2 = \frac{\omega_p^2}{\lambda} \left(\frac{m}{2\pi\kappa T_e} \right)^{1/2} \left[k_x^2 \int_0^\infty \sum_{n=1}^{\infty} 2n^2 \frac{\partial}{\partial \mu} (J_n^2(\mu)) \exp \left(-\frac{\mu^2}{2\lambda} \right) d\mu \int_{-\infty}^{+\infty} \frac{\exp \left(-\frac{mv_z^2}{2\kappa T_e} \right) dv_z}{(\omega - k_z v_z)^2 - (n\omega_c)^2} \right. \\ \left. + k_z^2 \int_0^\infty \sum_{n=0}^{\infty} \Delta_n J_n^2(\mu) \exp \left(-\frac{\mu^2}{2\lambda} \right) \mu d\mu \int_{-\infty}^{+\infty} \exp \left(-\frac{mv_z^2}{2\kappa T_e} \right) \left\{ \frac{(\omega - k_z v_z)^2 + (n\omega_c)^2}{[(\omega - k_z v_z)^2 - (n\omega_c)^2]^2} \right\} dv_z \right], \quad (25)$$

where λ has been written for $(k_x^2 \kappa T_e / m\omega_c^2)$ or $(k_x R)^2$; and hence, R is the radius of gyration of the average thermal electron.

The integration with respect to μ is accomplished using the identity [Bateman Manuscript Project, 1953]

$$\int_0^\infty J_n^2(\mu) \exp \left(-\frac{\mu^2}{2\lambda} \right) \mu d\mu = \lambda \exp(-\lambda) I_n(\lambda). \quad (26)$$

After some manipulation we arrive at the expression

$$k_x^2 + k_z^2 = \omega_p^2 \left(\frac{m}{2\pi\kappa T_e} \right)^{1/2} \left[k_x^2 \sum_{n=1}^{\infty} \left(\frac{2n^2}{\lambda} \right) \exp(-\lambda) I_n(\lambda) \int_{-\infty}^{+\infty} \frac{\exp\left(\frac{-mv_z^2}{2\kappa T_e}\right) dv_z}{(\omega - k_z v_z)^2 - (n\omega_c)^2} \right. \\ \left. + \left(\frac{m}{\kappa T_e} \right) \sum_{n=0}^{\infty} \Delta_n \exp(-\lambda) I_n(\lambda) \int_{-\infty}^{+\infty} \frac{k_z v_z (\omega - k_z v_z) \exp\left(\frac{-mv_z^2}{2\kappa T_e}\right) dv_z}{(\omega - k_z v_z)^2 - (n\omega_c)^2} \right]. \quad (27)$$

The integrand of (25) regarded as a function of v_R (or μ) has no poles. Consequently, no Landau damping terms appear in (27) due to the electron motions transverse to the magnetic field. The physical reason for their absence is that all electrons retrace their paths periodically, at the cyclotron frequency. Hence, there is no phase-mixing. However, (27) indicates that there can be Landau damping due to the velocity distribution in the z -direction, except for the special case of purely perpendicular propagation ($k_z=0$). Oblique propagation has been dealt with by Stix [1962] and we shall not discuss it further other than to remark that the damping will only be negligible if the condition $(\omega - n\omega_c) \gg k_z(2\kappa T_e/m)^{1/2}$ is satisfied. This implies that for modes corresponding to $n > 1$ propagation must be almost perpendicular to the magnetic field. By far the most interesting case, then, is that of perpendicular propagation which we shall now discuss in more detail.

2.5. Dispersion Relations for Perpendicular Propagation ($k_x \neq 0$, $k_z=0$)

For the two velocity distributions studied, (22) and (27) give

$$1 = \frac{\omega_p^2}{\omega_c^2} \sum_{n=1}^{\infty} \frac{\frac{\partial}{\partial \mu} (J_n^2(\mu))}{\left(\frac{\mu}{2} \right) \left[\left(\frac{\omega}{n\omega_c} \right)^2 - 1 \right]}, \quad 1 = \frac{\omega_p^2}{\omega_c^2} \sum_{n=1}^{\infty} \frac{\exp(-\lambda) I_n(\lambda)}{\left(\frac{\lambda}{2} \right) \left[\left(\frac{\omega}{n\omega_c} \right)^2 - 1 \right]}. \quad (28)$$

It may be remarked here that the series representation is not the most satisfactory for computation. An extremely useful integral expression is derived in an appendix to the paper.

The behavior of the two expressions of (28) differ considerably. The first is shown plotted in figures 2a, b, c, and d for different values of (ω_p^2/ω_c^2) . Each passband demonstrates a resonance ($k_x \rightarrow \infty$) at the appropriate cyclotron harmonic frequency. Cutoffs ($k_x \rightarrow 0$) occur at $(n+1)\omega_c$, and at the point X corresponding to the upper hybrid frequency $(\omega_p^2 + \omega_c^2)^{1/2}$ familiar from cold-plasma theory. The oscillatory nature of the dispersion relation in each passband stems from the nature of $\partial/\partial \mu [J_n^2(\mu)]$ which shows all of the zeros of $J_n(\mu)$ and $J'_n(\mu)$ as indicated in the figures. At low values of (ω_p^2/ω_c^2) individual modes do not couple. For larger values, such coupling can occur and gaps in the propagation appear. In these gaps the solution for ω obtained for real values of μ is complex, and indicates the existence of growing waves. This implies that the system is unstable [Crawford and Tataronis, 1965]. We shall return later to this extremely important point.

Figure 3 shows dispersion curves for the Maxwellian distribution. The same resonances and cutoffs are indicated as in figures 2a, b, c, and d but no coupling of the individual modes occurs and the system is absolutely stable.

So far, we have not discussed the effect of collisions on the otherwise undamped perpendicular propagation of cyclotron harmonic waves. It has been shown elsewhere [Crawford, Kino, and Weiss, 1964b] that electron/neutral collisions occurring with frequency ν can be taken into account in (28) by replacing ω by $(\omega - i\nu)$ and ω_p^2 by $\omega_p^2 \left(1 - i \frac{\nu}{\omega} \right)$. To obtain a simple order of magnitude estimate for the attenuation, it may be assumed that k_x is some function, K , of $(\omega - i\nu)$. Evaluating

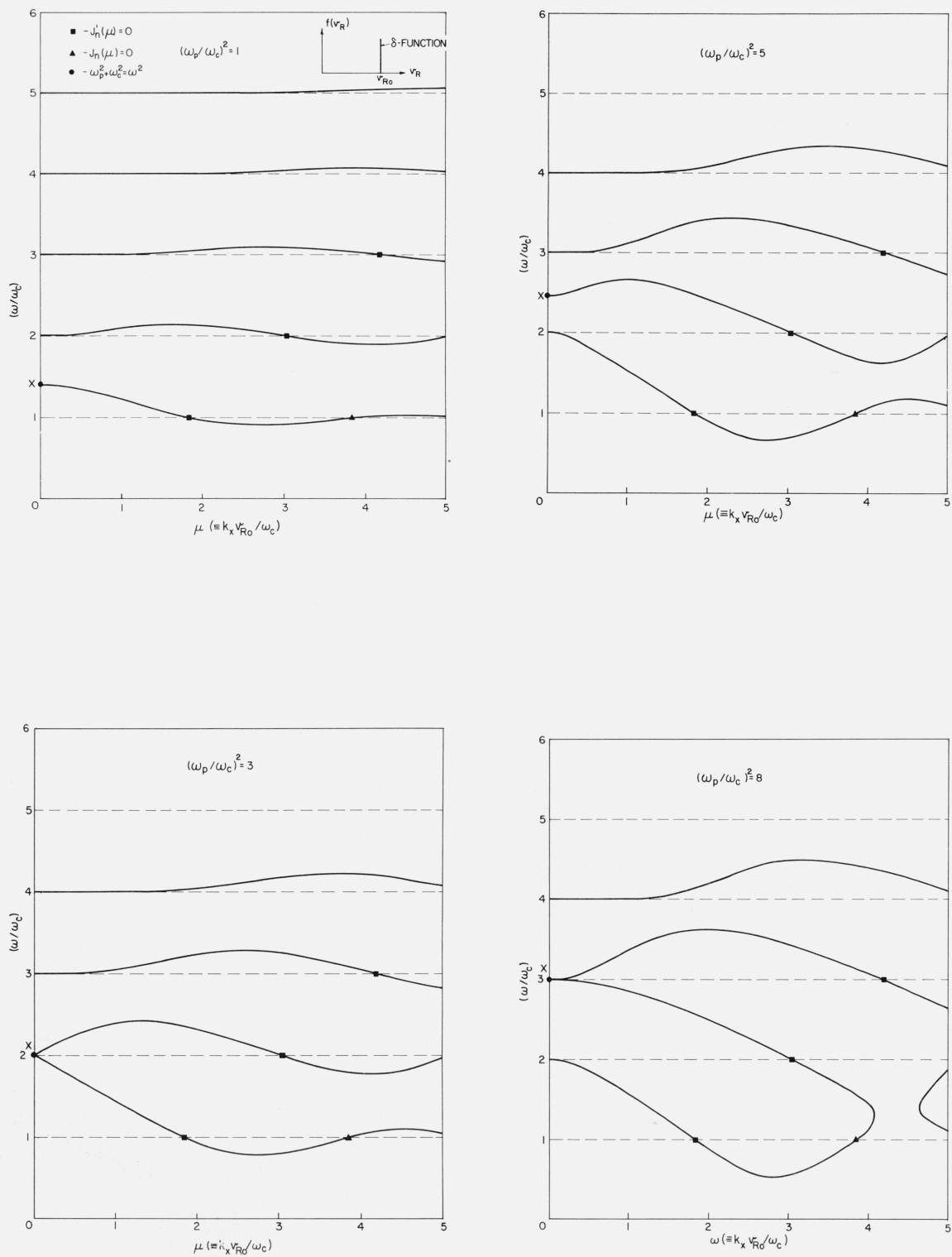
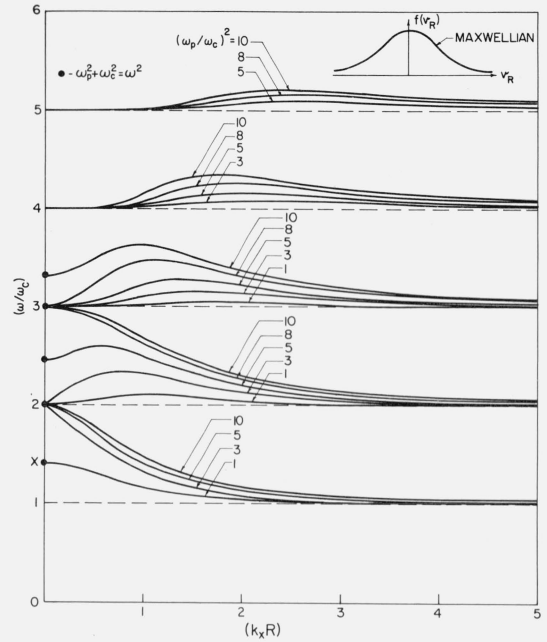


FIGURE 2. Dispersion characteristics for perpendicular propagation of cyclotron harmonic waves—monoenergetic transverse velocity distribution.

FIGURE 3. Dispersion characteristics for perpendicular propagation of cyclotron harmonic waves—Maxwellian transverse velocity distribution.



the real and imaginary parts of k_x , yields

$$k_{xr} + ik_{xi} \approx K(\omega) - i\nu \frac{\partial k_{xr}}{\partial \omega}. \quad (29)$$

The group velocity, v_g , may be introduced to write k_{xi} in the form $(-\nu/v_g)$ which represents attenuation/unit length. It is a feature of the computed curves of figures 2a, b, c, d, and 3 that v_g generally lies below the electron thermal velocity and decreases rapidly with increasing harmonic number. This means that the collision damping of perpendicular propagation may be strong if ν is not small, and will always limit the number of harmonics observable.

3. Experiments

3.1. General Comments

In this section we shall describe some of the experiments which confirm the existence of cyclotron harmonic waves. These are dealt with in detail elsewhere [Crawford, 1964b], and we shall only indicate briefly the nature of the phenomena involved. It should be emphasized that although the observations date back to 1959 [Wharton, 1960], their explanation was not then clear, and it was in 1963 [Canobbio and Croci, 1964; Tanaka, Mitani, and Kubo, 1963] that the suggestion was first advanced that these waves might be responsible for certain cyclotron harmonic noise phenomena in plasmas. Since these were the subject of the earliest studies we shall begin our brief review with some comments on them.

3.2. Studies of Plasma Emission

Although the first observations of radiation at $(n\omega_c)$ seem to have been those of Wharton [1960], in which noise emission from a pulsed mirror machine was studied, the first extensive series of experiments was that of Landauer [1961, 1962, 1964], using a PIG discharge as shown

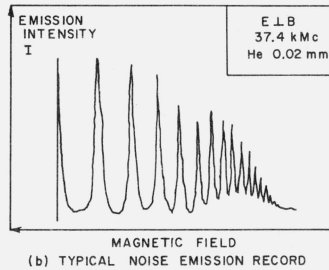
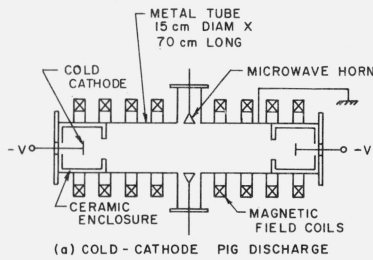


FIGURE 4. Cyclotron harmonic radiation from a PIG discharge [after Landauer, 1961, 1962, 1964].

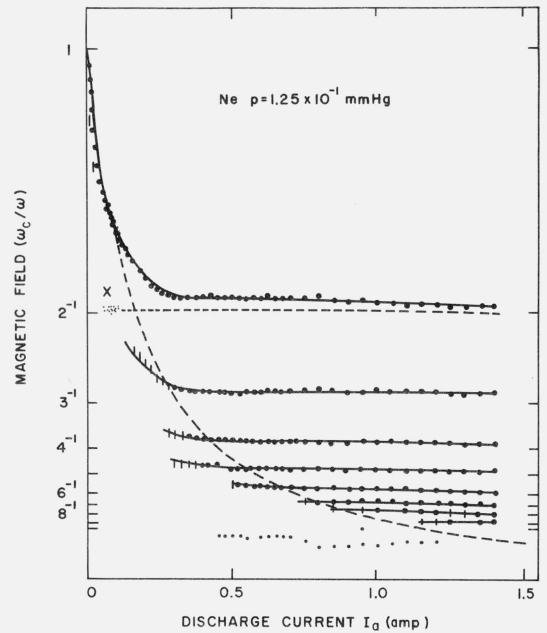


FIGURE 5. Dispersion characteristics of cyclotron harmonic radiation from a plasma [after Tanaka et al., 1963].

in figure 4. These indicated anomalously strong noise components up to the 45th harmonic, and fitting the relation $(\omega/\omega_c) = n$ to better than 2 percent. The most intense radiation was measured for the polarization $\vec{E} \perp \vec{B}$, but harmonic emission was also obtained for $\vec{E} \parallel \vec{B}$. We may remark here that this latter result may easily be predicted from examination of the dielectric tensor obtained without making the quasi-static assumption. This indicates that the ordinary wave, $\vec{E} \parallel \vec{B}$, $\vec{k} \perp \vec{B}$ should also show cyclotron harmonic resonances [Stix, 1962; Kubo et al., 1964].

The major conclusions of Landauer have been confirmed by Dreicer [1964], in a PIG discharge, by Lustig et al. [1964]; Mitani and Kubo [1961]; Tanaka et al. [1963a, b], and Mitani et al. [1964a, b]. These latter worked with hot-cathode discharges and were the first to remark that there are significant discrepancies from the relation $(\omega/\omega_c) = n$ for maximum noise emission. Typical results are given in figure 5.

The reasons for the very high noise production in PIG discharges are clear from the discussion of section 2.5. Such a discharge contains streams of electrons having energy equal to the operating voltage, and presents ideal conditions for transverse instabilities or strong beam/plasma interactions with the cyclotron harmonic modes.

It might be argued that strong directional velocity components were not present in some hot-cathode discharge experiments where many harmonics were also observed [Lustig et al., 1964; Mitani and Kubo, 1961; Tanaka and Kubo, 1964; Kubo et al., 1964]. It was always noted in these studies, however, that noise emission was highest near the cathode end of the tube and at points where the plasma entered the magnetic field. The results imply, then, that streams of primary electrons entering from the cathode region, and acquiring high transverse velocity components, are effective in producing cyclotron harmonic radiation.

As further support for the hypothesis concerning the effectiveness of charged particle beams in exciting cyclotron harmonic resonances, we may refer to radiation measurements carried out in the millimeter pressure range where such streams would be rapidly dispersed. Experimental results along these lines were reported by Bekefi et al. [1961a, b; Fields et al., 1961; Hirshfield and Brown, 1961; Brown and Bekefi, 1962] and have been substantially confirmed by Tanaka and

Kubo [1964]. They show characteristic noise radiation dependent on the gas: in helium and hydrogen, the effective radiation temperature corresponds to the electron temperature, but in neon, argon, and xenon narrow peaks in the radiation temperature occur, centered on ω_c . These results can be ascribed to strongly non-Maxwellian electron velocity distributions in the last-mentioned gases, where the electron-neutral collision cross section decreases with energy in the energy range appropriate to the measurements [Ohara, 1962, 1963]. The conclusion to be drawn, then, is that beam-type or "humped" velocity distributions are capable of exciting strong cyclotron harmonic waves.

The best test of this, of course, is to use an electron beam to produce the plasma. This ensures the presence of both beam and plasma. Two experiments along these lines have been reported [Bekefi and Hooper, 1964; Gruber et al., 1964]. In the first of these, the beam and the discharge tube were coaxial with the magnetic field, and the gun was only partially immersed, so that about 15 percent of the longitudinal beam energy was converted into transverse rotation. Emission at ω_c due to the beam electrons alone could be observed, appropriately Doppler-shifted in frequency. When mercury-vapor was present, however, the radiated signal increased by 35 dB and similar Doppler-shifted components could be seen up to the fourth harmonic. Radiation from a beam-produced plasma in helium up to the 15th harmonic was studied by Gruber et al. [1964], who examined the radial and longitudinal amplitude variations of the output signals by means of a movable probe. They presented results showing fast-wave growth of the 8th to 10th cyclotron harmonics at about 11 dB per wavelength in the direction of the beam, and with rapid radial fall-off.

3.3. Absorption Studies

Having discussed experiments in which noise emission only has been studied, we now turn to attempts to excite resonance behavior with an external signal. Bekefi et al. [1962] reported work on a hot-cathode helium discharge in a cylindrical cavity excited in the TE_{011} mode, and immersed in an axial magnetic field. It was found that for $\omega_p^2 \ll \omega^2$, resonant absorption occurred at ω_c , and that for $\omega_p^2 \approx \omega^2$, weak absorption of up to the first eight cyclotron harmonics occurred.

Comparisons of absorption and emission under identical experimental conditions have been made by Tanaka and Kubo [1964]. Weak absorption occurred up to the 5th harmonic, whereas noise radiation was appreciable up to the 8th harmonic. This is to be expected since we are dealing with radiation orders of magnitude higher than that appropriate to thermal equilibrium. Reciprocity between emission and absorption would occur only under these special conditions.

Further absorption studies, both theoretical and experimental, have been made by Buchsbaum and Hasegawa [1964] to explain an effect first remarked by Kubo et al. [1964]: the noise radiation can show series of subsidiary resonances lying between the main harmonic resonance. With increasing discharge current, these successively approach the harmonic resonances, increasing in magnitude as they do so, and disappear on reaching them. The subsidiary resonances were explained as being due to the presence of radial electron density variation in the plasma. They are analogous to the Tonks-Dattner resonances mentioned in section 1, but are "trapped" in the center of the column, rather than in the region close to the wall. An alternative source of subsidiary resonances due to wave interference has been studied recently by Harp [1965].

By far the most interesting observations from the point of view of ionospheric propagation were the results obtained with the "Alouette" satellite [Calvert and Goe, 1963; Lockwood, 1963]. In these, the vehicle carried a transmitter/antenna system giving a pulsed variable frequency signal. Strong resonances were found to occur within 1 percent of the calculated local cyclotron harmonic frequencies, up to about the 10th cyclotron harmonic. Prolonged ringing was observed at these frequencies, persisting for many periods after the end of the transmitter pulse. Since no such effect has been seen from the ground, we may deduce that the effect was local to the transmitter.

It was suggested that since the perpendicular propagation of cyclotron harmonic modes is not Landau damped, and collision frequencies in the ionosphere are relatively low, a ringing

effect is to be expected near $n\omega_c$, where the group velocity is low and the energy is not transported rapidly away from the transmitter [Crawford, Kino, and Weiss, 1964a]. This explanation has been developed by Fejer and Calvert [1964], and Sturrock [1965], who have estimated the damping rate of the resonances.

3.4. Propagation Studies

Although the results described so far provide strong support for the cyclotron harmonic wave picture, in that they are all consistent with waves propagating nearly normal to the magnetic field, one very important observation is lacking. This is a direct measurement of transmission across the magnetic field between two antennas. Such an experiment was carried out for the first time in 1964 [Crawford, Kino, and Weiss, 1964a, 1964b]. In the original experiment two Langmuir probes, parallel to each other and to a static magnetic field, and situated a few millimeters apart in a hot-cathode mercury-vapor discharge were used as emitting and receiving antennas respectively for a 400 MHz signal. Peaks in the received signal were observed for certain combinations of discharge current and magnetic field. Typical data are shown in figure 6a.

A direct comparison with the theoretical curves of figure 3 cannot be made since ω_c and ω_p (proportional to $I_a^{1/2}$) were the experimental variables. It is instructive, however, to compute from the theoretical dispersion relation the variation of (ω_c^2/ω^2) with (ω_p^2/ω^2) . This is indicated in figures 7a, b, c. The coupling of the cyclotron harmonic frequencies and the cold plasma upper hybrid resonance are clearly indicated as the parameter (v_p/v_T) is varied. There is striking agreement between the general shapes of these curves and the experimental curves of figures 5 and 6a.

If the radial wavelength were constant in the experiment the experimental points could be compared directly to the theory. However, the nonuniform density in the cylindrical plasma column employed causes the modes to be trapped internally so that the radial wavelength varies with current. This is clearly indicated by the presence of two distinct sets of resonances in figure 6a. These are for two different trapped modes. When a more uniform discharge is produced by rf the transmission resonances are much cleaner. A typical record is shown in figure 6b and indicates the presence of only one set of peaks. We can, however, indicate on the plot of figure 6a the approximate location of the upper hybrid resonance frequency obtained by an independent method [Crawford et al., 1963]. This is shown dotted in figure 6a, and reinforces our belief in the validity of the cyclotron harmonic wave explanation of the transmission resonances.

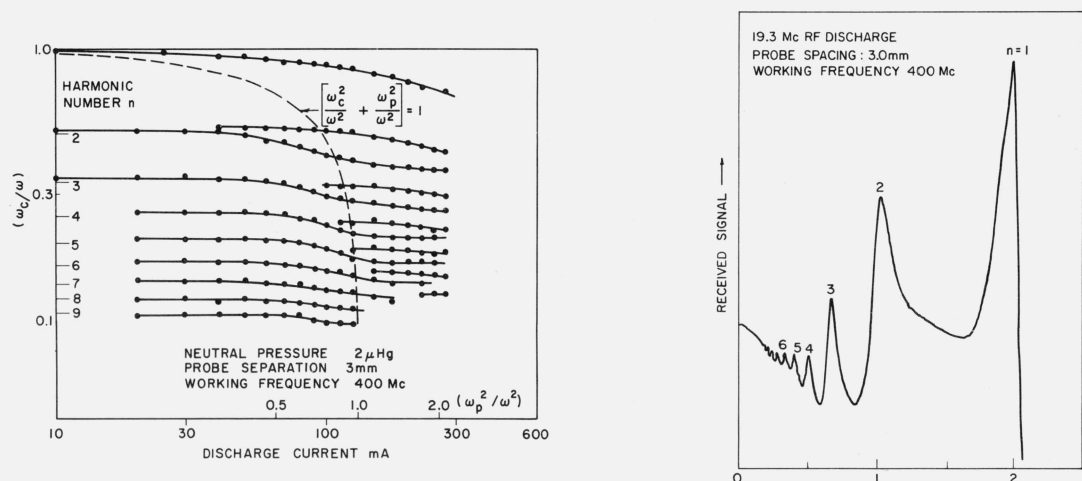


FIGURE 6. *Experimental observations of perpendicular propagation.*

- (a) Dispersion characteristics [after Crawford, Kino, and Weiss 1964].
- (b) Transmission record for an rf discharge (the abscissa may be calibrated in terms of (ω_c/ω)).

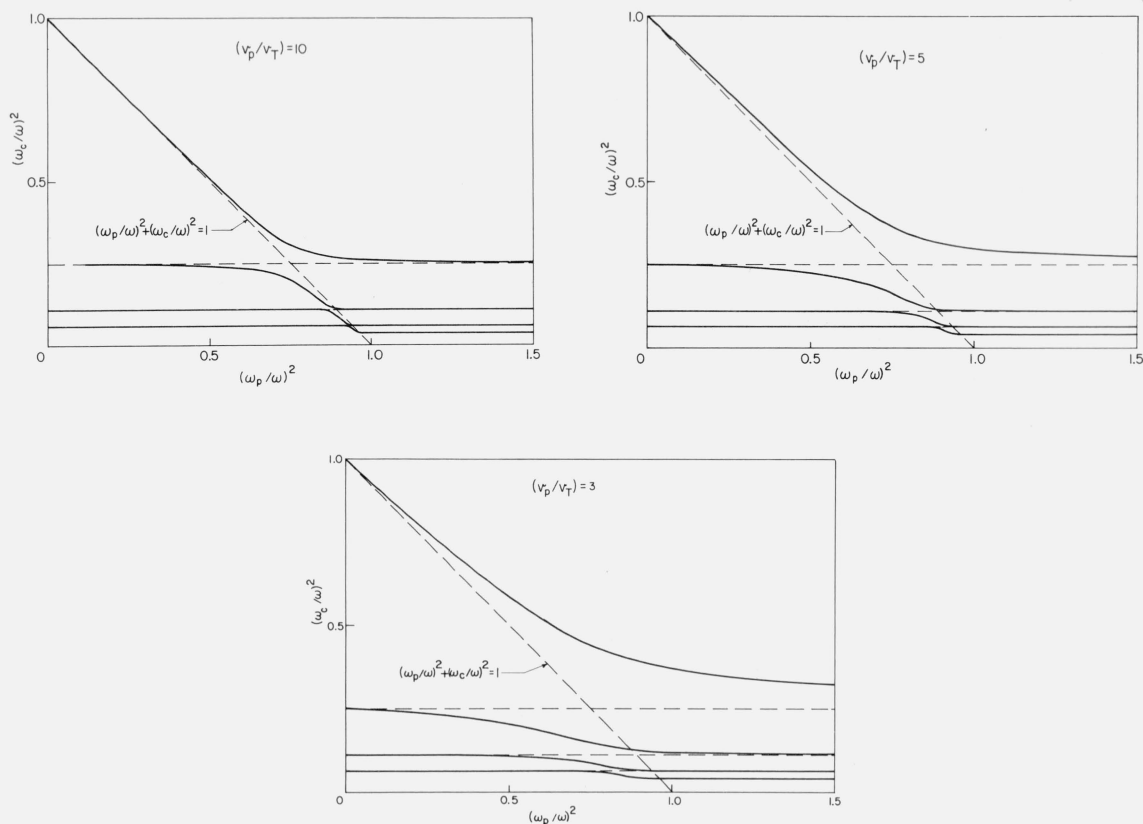


FIGURE 7. Alternative representation of the perpendicular propagation dispersion characteristics of figure 3 for a Maxwellian transverse velocity distribution [$(v_p = (\omega/k_x)$, $v_T = (\kappa T_e/m)^{1/2}$].

4. Discussion

The complicated single-particle analysis required to demonstrate the existence of cyclotron harmonic waves has probably been partly responsible for the fact that their existence has been established experimentally almost by accident, rather than as the result of deliberate searches. Of the studies described, only those concerned with transmission were made specifically to compare with the form of the theoretical dispersion relation.

Now that their existence has been established, the importance of these waves can be expected to grow. First, there is the possibility of applying them in practical laboratory microwave amplifiers and oscillators [Crawford and Kino, 1964]. As they are slow waves, interaction with an electron beam directed either along the magnetic field, or across it, will cause spatially growing waves. Alternatively, a beam with internal motions can interact directly with slow or fast wave circuits to produce gain. Such a beam could be produced in nature by a charged particle stream entering a region of increasing magnetic field. Second, taking the opposite point of view, the explanation of many plasma noise phenomena, including extraterrestrial cyclotron emission, may be better understood in terms of cyclotron harmonic wave instabilities [Twiss, 1958; Schneider, 1959; Bekefi, Hirshfield, and Brown 1961a and 1961b]. A third region of application is in diagnostics. The "Alouette" ringing experiments suggests that the resonances could be used for measuring magnetic field accurately. Attenuation measurements could be used to determine collision frequencies, and in principle it should be possible to determine electron density and temperature from transmission measurements between two antennas.

All that has been said in this paper about electron effects can be extended to the positive ions. Consequently, it is not surprising that experimental observations have already been made of ion cyclotron harmonic emission from thermonuclear fusion study plasmas [Bazhanova et al., 1962; Pistunovich and Shafranov, 1961; Crawford, 1964c]. The possibility that ion cyclotron waves might be useful for plasma heating immediately suggests itself.

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5. Appendix. Integral Representation of the Dispersion Relation for Perpendicular Propagation ($k_z=0$)

A convenient integral form for computation may be derived from (17), (20), and (21) by carrying out the velocity integrations first. This leads to

$$1 = \frac{\omega_p^2}{\omega_c^2} \int_{-\infty}^0 \exp i\Omega\theta \sin \theta J_0 \left(2\mu \sin \frac{\theta}{2} \right) d\theta \quad (30)$$

where θ has been written for $(\omega_c\tau)$, and Ω for (ω/ω_c) . Alternatively, this can be expressed as the summation

$$1 = \frac{\omega_p^2}{\omega_c^2} \sum_{n=0}^{\infty} \int_{-2\pi(n+1)}^{-2\pi n} \exp i\Omega\theta \sin \theta J_0 \left(2\mu \sin \frac{\theta}{2} \right) d\theta. \quad (31)$$

Substitution of $[\varphi - 2\pi(n+1)]$ for θ reduces the summation to

$$1 = \frac{\omega_p^2}{\omega_c^2} \sum_{n=0}^{\infty} (\exp [-i2\pi\Omega])^{n+1} \int_0^{2\pi} \exp (i\Omega\varphi) \sin \varphi J_0 \left(2\mu \sin \frac{\varphi}{2} \right) d\varphi. \quad (32)$$

If the presence of a small loss component is assumed, this can be further simplified to

$$\exp [i2\pi\Omega] - 1 = \frac{\omega_p^2}{\omega_c^2} \int_0^{2\pi} \exp (i\Omega\varphi) \sin \varphi J_0 \left(2\mu \sin \frac{\varphi}{2} \right) d\varphi, \quad (33)$$

which separates into the two forms,

$$\begin{aligned} \cos 2\pi\Omega &= 1 + \frac{\omega_p^2}{\omega_c^2} \int_0^{2\pi} \cos \Omega\varphi \sin \varphi J_0 \left(2\mu \sin \frac{\varphi}{2} \right) d\varphi \\ \sin 2\pi\Omega &= \frac{\omega_p^2}{\omega_c^2} \int_0^{2\pi} \sin \Omega\varphi \sin \varphi J_0 \left(2\mu \sin \frac{\varphi}{2} \right) d\varphi, \end{aligned} \quad (34)$$

either of which can be used for computation.

These can be regarded as general formulas from which integral expressions for velocity distributions other than the δ -function can be deduced. As an example, for the Maxwellian of (24) we must evaluate the weighting integral

$$\left(\frac{m}{2\pi\kappa T_e} \right) \int_0^{\infty} J_0 \left(2\mu \sin \frac{\varphi}{2} \right) \exp \left(-\frac{mv_R^2}{2\kappa T_e} \right) 2\pi v_R dv_R. \quad (35)$$

The Bessel function identity required is [Bateman Manuscript Project, 1953]

$$\int_0^\infty J_0(2\mu \sin \frac{\varphi}{2}) \exp\left(-\frac{\mu^2}{2\lambda}\right) \mu d\mu \equiv \lambda \exp\left(-2\lambda \sin^2 \frac{\varphi}{2}\right). \quad (36)$$

The dispersion relation is consequently given by either of the expressions

$$\begin{aligned} \cos 2\pi\Omega &= 1 + \frac{\omega_p^2}{\omega_c^2} \exp(-\lambda) \int_0^{2\pi} \cos \Omega\varphi \sin \varphi \exp(\lambda \cos \varphi) d\varphi \\ \sin 2\pi\Omega &= \frac{\omega_p^2}{\omega_c^2} \exp(-\lambda) \int_0^{2\pi} \sin \Omega\varphi \sin \varphi \exp(\lambda \cos \varphi) d\varphi. \end{aligned} \quad (37)$$

The integral relations can be used to determine the series forms of (28) if appropriate Bessel function identities are used. For example, differentiation of (18) with respect to μ yields

$$\sin \varphi J_0(2\mu \sin \frac{\varphi}{2}) \equiv \sum_{n=1}^{\infty} \frac{2n}{\mu} \frac{\partial}{\partial \mu} [J_n^2(\mu)] \sin n\theta, \quad (38)$$

which can be substituted in (34) and integrated term by term. Similarly for the Maxwellian distribution we have [Bateman Manuscript Project, 1953]

$$\exp(\lambda \cos \varphi) \equiv \sum_{n=0}^{\infty} \cos n\varphi I_n(\lambda), \quad (39)$$

which can be differentiated and substituted in (37). Integration, term by term, then gives (28).

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